

# Using Assumptions in Gentzen-type Systems

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## Abstract

The consequence relation between formulas which is induced by a fully structural Gentzen-type system  $G$  is usually taken to be:  $\Gamma \vdash_G \psi$  iff the sequent  $\Gamma \Rightarrow \psi$  is provable in  $G$ . However, no less useful is the consequence relation  $\vdash_G^v$  defined by:  $\Gamma \vdash_G^v \psi$  iff the sequent  $\Rightarrow \psi$  is derivable in  $G$  from the set of sequents  $\{\Rightarrow \varphi \mid \varphi \in \Gamma\}$ . This is one particular case in which it is useful to infer a sequent from a set of assumptions where these assumptions are again sequents. We present several other examples of the usefulness of such inferences, including non-classical cases in which doing this is not only useful, but really essential for the expressive power of the logic. The main technical tool used in the various applications we present is a generalization of the usual cut-elimination theorem (which treats only assumptions-free derivations) to what we call the *strong cut-elimination theorem* (which applies also to derivations of sequents from other sequents).