

**Every proof is (and isn't) a dialogue:  
On the dialogical foundations of logic**

Catarina Dutilh Novaes  
ILLC and Department of Philosophy  
University of Amsterdam

# Introduction

- What are the connections between dialogues and proofs?
- This question can be addressed technically, but also conceptually, and even historically.
- *Different* dialogical conceptions of proof and logic seem to be available.
- Here, I present the BIO conception: built-in opponent.
- We can then go back to the formalisms to see how faithfully they represent the different dialogical conceptions of proof.

# Introduction

- What are the connections between dialogues and proofs?
- This question can be addressed technically, but also conceptually, and even historically.
- *Different* dialogical conceptions of proof and logic seem to be available.
- Here, I present the BIO conception: built-in opponent.
- We can then go back to the formalisms to see how faithfully they represent the different dialogical conceptions of proof.

# Introduction

- What are the connections between dialogues and proofs?
- This question can be addressed technically, but also conceptually, and even historically.
- *Different* dialogical conceptions of proof and logic seem to be available.
- Here, I present the BIO conception: built-in opponent.
- We can then go back to the formalisms to see how faithfully they represent the different dialogical conceptions of proof.

# Introduction

- What are the connections between dialogues and proofs?
- This question can be addressed technically, but also conceptually, and even historically.
- *Different* dialogical conceptions of proof and logic seem to be available.
- Here, I present the **BIO** conception: **built-in opponent**.
- We can then go back to the formalisms to see how faithfully they represent the different dialogical conceptions of proof.

# Introduction

- What are the connections between dialogues and proofs?
- This question can be addressed technically, but also conceptually, and even historically.
- *Different* dialogical conceptions of proof and logic seem to be available.
- Here, I present the BIO conception: built-in opponent.
- We can then go back to the formalisms to see how faithfully they represent the different dialogical conceptions of proof.

## Plan of the talk

- The forgotten dialogical origins of logic
- Proofs as dialogues
- Different formalisms for a dialogical conception of proofs

## The forgotten dialogical origins of logic

- Plato's dialogues and the pre-Socratic dialogical method (Marion & Castelnerac 2009)
- Aristotle's 'older' logical texts (*Topics* and *Sophistical Refutations*) are explicitly about debating.
- Syllogistic is less obviously about debating, but: "A deduction is a discourse..."
- Latin Middle Ages: *logica = dialectica*
- Domingo de Soto (16<sup>th</sup> century): "Dialectic is the art or science of disputing".



# The forgotten dialogical origins of logic

- Plato's dialogues and the pre-Socratic dialogical method (Marion & Castelnerac 2009)
- Aristotle's 'older' logical texts (*Topics* and *Sophistical Refutations*) are explicitly about debating.
- Syllogistic is less obviously about debating, but: "A deduction is a discourse..."
- Latin Middle Ages: *logica = dialectica*
- Domingo de Soto (16<sup>th</sup> century): "Dialectic is the art or science of disputing".

## The forgotten dialogical origins of logic

- Plato's dialogues and the pre-Socratic dialogical method (Marion & Castelnerac 2009)
- Aristotle's 'older' logical texts (*Topics* and *Sophistical Refutations*) are explicitly about debating.
- Syllogistic is less obviously about debating, but: "A deduction is a discourse..."
- Latin Middle Ages: *logica = dialectica*
- Domingo de Soto (16<sup>th</sup> century): "Dialectic is the art or science of disputing".

## The forgotten dialogical origins of logic

- Plato's dialogues and the pre-Socratic dialogical method (Marion & Castelnerac 2009)
- Aristotle's 'older' logical texts (*Topics* and *Sophistical Refutations*) are explicitly about debating.
- Syllogistic is less obviously about debating, but: "A deduction is a discourse..."
- Latin Middle Ages: *logica = dialectica*
- Domingo de Soto (16<sup>th</sup> century): "Dialectic is the art or science of disputing".

## The forgotten dialogical origins of logic

- Plato's dialogues and the pre-Socratic dialogical method (Marion & Castelnerac 2009)
- Aristotle's 'older' logical texts (*Topics* and *Sophistical Refutations*) are explicitly about debating.
- Syllogistic is less obviously about debating, but: "A deduction is a discourse..."
- Latin Middle Ages: *logica = dialectica*
- Domingo de Soto (16<sup>th</sup> century): "Dialectic is the art or science of disputing".

## The birth of deduction in Greek mathematics

“Greek mathematics reflects the importance of persuasion. It reflects the role of orality, in the use of formulae, in the structure of proofs ... But this orality is regimented into a written form, where vocabulary is limited, presentations follow a relatively rigid pattern... It is at once oral and written...” (Netz 1999, 297/8)

- The deductive method emerged as an approach to argumentation (against e.g. Sophists).
- **A proof is and isn't a dialogue:** a hybrid entity between orality and writing.

## The birth of deduction in Greek mathematics

“Greek mathematics reflects the importance of persuasion. It reflects the role of orality, in the use of formulae, in the structure of proofs ... But this orality is regimented into a written form, where vocabulary is limited, presentations follow a relatively rigid pattern... It is at once oral and written...” (Netz 1999, 297/8)

- The deductive method emerged as an approach to argumentation (against e.g. Sophists).
- **A proof is and isn't a dialogue:** a hybrid entity between orality and writing.

## The birth of deduction in Greek mathematics

“Greek mathematics reflects the importance of persuasion. It reflects the role of orality, in the use of formulae, in the structure of proofs ... But this orality is regimented into a written form, where vocabulary is limited, presentations follow a relatively rigid pattern... It is at once oral and written...” (Netz 1999, 297/8)

- The deductive method emerged as an approach to argumentation (against e.g. Sophists).
- **A proof is and isn't a dialogue:** a hybrid entity between orality and writing.

## When logic abandoned its dialogical origins

“After that, he should study logic. I do not mean the logic of the Schools, for this is strictly speaking nothing but a dialectic which teaches *ways of expounding to others what one already knows* or even of holding forth without judgment about things one does not know. Such logic corrupts good sense rather than increasing it. I mean instead the kind of logic which teaches us to direct our reason with a view to discovering the truths of which we are ignorant.” (Preface to French edition of the *Principles*, in (Descartes 1988, 186))



## Kant and the internalization of logic

- Kant transformed the very conception of logic, from the point of view of transcendental idealism (Longuenesse 1998).
- He selectively absorbed and transformed concepts such as ‘judgment’ and ‘categories’.

The laws of general logic are “*without content and merely formal*”; general logic “. . . *abstracts from all content of knowledge . . . and . . . treats of the form of thought in general.*” (KrV: A152/B19)

## Kant and the internalization of logic

- Kant transformed the very conception of logic, from the point of view of transcendental idealism (Longuenesse 1998).
- He selectively absorbed and transformed concepts such as ‘judgment’ and ‘categories’.

The laws of general logic are “*without content and merely formal*”; general logic “. . . *abstracts from all content of knowledge . . . and . . . treats of the form of thought in general.*” (KrV: A152/B19)

## Kant and the internalization of logic

- Kant transformed the very conception of logic, from the point of view of transcendental idealism (Longuenesse 1998).
- He selectively absorbed and transformed concepts such as ‘judgment’ and ‘categories’.

The laws of general logic are “*without content and merely formal*”; general logic “. . . *abstracts from all content of knowledge . . . and . . . treats of the form of thought in general.*” (KrV: A152/B19)

## **2. Proofs as dialogues**

## Proofs as discourse and justification

- A demonstration (proof) is a discourse aimed at compelling the audience to accept (the truth of) the conclusion, if they accept (the truth of) the premises.
- Contrast with calculation: a calculation is for ‘individual consumption’; a demonstration, a proof, is intended for *others*.
- In Chinese mathematics, there is a predominance of focus on calculations and algorithms, but occasionally there are proofs of their correctness in *justificatory contexts* (e.g. commentaries).

## Proofs as discourse and justification

- A demonstration (proof) is a discourse aimed at compelling the audience to accept (the truth of) the conclusion, if they accept (the truth of) the premises.
- Contrast with calculation: a calculation is for ‘individual consumption’; a demonstration, a proof, is intended for *others*.
- In Chinese mathematics, there is a predominance of focus on calculations and algorithms, but occasionally there are proofs of their correctness in *justificatory contexts* (e.g. commentaries).

## Proofs as discourse and justification

- A demonstration (proof) is a discourse aimed at compelling the audience to accept (the truth of) the conclusion, if they accept (the truth of) the premises.
- Contrast with calculation: a calculation is for ‘individual consumption’; a demonstration, a proof, is intended for *others*.
- In Chinese mathematics, there is a predominance of focus on calculations and algorithms, but occasionally there are proofs of their correctness in *justificatory contexts* (e.g. commentaries).

## Proofs as adversarial dialogues

- As the historical origins of deduction show, it is a rather contrived form of dialogical interaction.
- Proofs as **adversarial** dialogues. The participants have opposite goals: establishing the conclusion vs. blocking the establishment of the conclusion.
- Socrates as the opponent: the one constantly looking for flaws (in particular inconsistencies) in the argumentation put forward by proponent in order to argue for the thesis.



## Proofs as adversarial dialogues

- As the historical origins of deduction show, it is a rather contrived form of dialogical interaction.
- Proofs as **adversarial** dialogues. The participants have opposite goals: establishing the conclusion vs. blocking the establishment of the conclusion.
- Socrates as the opponent: the one constantly looking for flaws (in particular inconsistencies) in the argumentation put forward by proponent in order to argue for the thesis.

## Proofs as adversarial dialogues

- As the historical origins of deduction show, it is a rather contrived form of dialogical interaction.
- Proofs as **adversarial** dialogues. The participants have opposite goals: establishing the conclusion vs. blocking the establishment of the conclusion.
- Socrates as the opponent: the one constantly looking for flaws (in particular inconsistencies) in the argumentation put forward by proponent in order to argue for the thesis.

## The ‘built-in’ opponent (BIO) conception

- What is the essence of mathematical argumentation (demonstration)?
- At every deductive step (which must be perspicuous –Wittgenstein), there should be no counterexamples.
- The role of opponent is to look for counterexamples, i.e. situations where the premises hold but the conclusion (of each individual step) does not.
- The deductive method has **internalized** the opponent, it is now built into the framework: every inferential step must be immune to counterexamples.

## The ‘built-in’ opponent (BIO) conception

- What is the essence of mathematical argumentation (demonstration)?
- At every deductive step (which must be perspicuous –Wittgenstein), there should be no counterexamples.
- The role of opponent is to look for counterexamples, i.e. situations where the premises hold but the conclusion (of each individual step) does not.
- The deductive method has **internalized** the opponent, it is now built into the framework: every inferential step must be immune to counterexamples.

## The ‘built-in’ opponent (BIO) conception

- What is the essence of mathematical argumentation (demonstration)?
- At every deductive step (which must be perspicuous –Wittgenstein), there should be no counterexamples.
- The role of opponent is to look for counterexamples, i.e. situations where the premises hold but the conclusion (of each individual step) does not.
- The deductive method has **internalized** the opponent, it is now built into the framework: every inferential step must be immune to counterexamples.

## The ‘built-in’ opponent (BIO) conception

- What is the essence of mathematical argumentation (demonstration)?
- At every deductive step (which must be perspicuous –Wittgenstein), there should be no counterexamples.
- The role of opponent is to look for counterexamples, i.e. situations where the premises hold but the conclusion (of each individual step) does not.
- The deductive method has **internalized** the opponent, it is now built into the framework: every inferential step must be immune to counterexamples.

## Indefeasibility and monotonicity

- A *winning strategy* for proponent consists in a sequence of inferential steps to which there are no counterexamples: **indefeasible** steps.
- No matter what external information opponent brings in, in a winning strategy it will not defeat the individual inferential steps.
- Hence, monotonicity is easily accounted for in terms of indefeasibility.

## Indefeasibility and monotonicity

- A *winning strategy* for proponent consists in a sequence of inferential steps to which there are no counterexamples: **indefeasible** steps.
- No matter what external information opponent brings in, in a winning strategy it will not defeat the individual inferential steps.
- Hence, monotonicity is easily accounted for in terms of indefeasibility.



## Indefeasibility and monotonicity

- A *winning strategy* for proponent consists in a sequence of inferential steps to which there are no counterexamples: **indefeasible** steps.
- No matter what external information opponent brings in, in a winning strategy it will not defeat the individual inferential steps.
- Hence, monotonicity is easily accounted for in terms of indefeasibility.

## The BIO conception of proofs as a hybrid entity

- The conception of a (deductive) proof emerged against the background of certain dialogical practices, but it was then modified and regimented.
- In particular, transformation from *oral* to *written*.
- The opponent has been *internalized* (built-in) by the proof method, no longer playing the same active role.
- Thus, proofs are no longer dialogues in the ‘proper’ sense of the term.

## The BIO conception of proofs as a hybrid entity

- The conception of a (deductive) proof emerged against the background of certain dialogical practices, but it was then modified and regimented.
- In particular, transformation from *oral* to *written*.
- The opponent has been *internalized* (built-in) by the proof method, no longer playing the same active role.
- Thus, proofs are no longer dialogues in the ‘proper’ sense of the term.

## The BIO conception of proofs as a hybrid entity

- The conception of a (deductive) proof emerged against the background of certain dialogical practices, but it was then modified and regimented.
- In particular, transformation from *oral* to *written*.
- The opponent has been *internalized* (built-in) by the proof method, no longer playing the same active role.
- Thus, proofs are no longer dialogues in the ‘proper’ sense of the term.

## The BIO conception of proofs as a hybrid entity

- The conception of a (deductive) proof emerged against the background of certain dialogical practices, but it was then modified and regimented.
- In particular, transformation from *oral* to *written*.
- The opponent has been *internalized* (built-in) by the proof method, no longer playing the same active role.
- Thus, proofs are no longer dialogues in the ‘proper’ sense of the term.

### **3. Different formalisms for the BIO dialogical conception of proofs**

## Hodges' 'Dawkins question'

“If we want P's motivation in a game  $G$  to have any explanatory value, then we need to understand what is achieved if P does win. In particular we should be able to tell a realistic story of a situation in which some agent called P is trying to do something intelligible, and doing it is the same thing as winning in the game.” (Hodges, SEP entry on logic and games)

- Who plays the game, and why do they play it? What are the goals of the players?

## Different formalisms and the Dawkins question

- Hodges claims that many game-based logical formalisms fail to provide appropriate answers to the Dawkins question.
- They fail to define the goals and motivations of the players in a satisfactory way, defining “games that nobody plays”.
- In particular, dialogical logics, in their regimented nature, and while respecting the ‘agonistic’ origins of logic (Krabbe), end up too far away from actual dialogues.



## Different formalisms and the Dawkins question

- Hodges claims that many game-based logical formalisms fail to provide appropriate answers to the Dawkins question.
- They fail to define the goals and motivations of the players in a satisfactory way, defining “games that nobody plays”.
- In particular, dialogical logics, in their regimented nature, and while respecting the ‘agonistic’ origins of logic (Krabbe), end up too far away from actual dialogues.

## Different formalisms and the Dawkins question

- Hodges claims that many game-based logical formalisms fail to provide appropriate answers to the Dawkins question.
- They fail to define the goals and motivations of the players in a satisfactory way, defining “games that nobody plays”.
- In particular, dialogical logics, in their regimented nature, and while respecting the ‘agonistic’ origins of logic (Krabbe), end up too far away from actual dialogues.

## Which formalism for the BIO conception of proof?

- I submit that, of the different proof formalisms available, **natural deduction systems** come closest to capturing the idea of a built-in opponent.
- It is a mistake to think of their ‘naturalness’ in terms of our ‘natural reasoning patterns’ (inner, mono-agent process) rather than in terms of arguing patterns (public, multi-agent situations).
- Arguably, what is ‘natural’ about natural deduction is the way it reflects how we argue (deductively) to establish a conclusion from premises.

## Which formalism for the BIO conception of proof?

- I submit that, of the different proof formalisms available, **natural deduction systems** come closest to capturing the idea of a built-in opponent.
- It is a mistake to think of their ‘naturalness’ in terms of our ‘natural reasoning patterns’ (inner, mono-agent process) rather than in terms of arguing patterns (public, multi-agent situations).
- Arguably, what is ‘natural’ about natural deduction is the way it reflects how we argue (deductively) to establish a conclusion from premises.

## Which formalism for the BIO conception of proof?

- I submit that, of the different proof formalisms available, **natural deduction systems** come closest to capturing the idea of a built-in opponent.
- It is a mistake to think of their ‘naturalness’ in terms of our ‘natural reasoning patterns’ (inner, mono-agent process) rather than in terms of arguing patterns (public, multi-agent situations).
- Arguably, what is ‘natural’ about natural deduction is the way it reflects how we argue (deductively) to establish a conclusion from premises.

## Conclusions

- Historical and conceptual analysis can unearth some of the connections between the concepts of dialogues and proofs.
- It suggests that every proof can be understood as a ‘dialogue’ between the one formulating it and a built-in opponent: the BIO conception.
- This conception can serve as a starting point to evaluate, from a philosophical perspective, different dialogue-based logical formalisms.
- It offers a possible answer to the Dawkins question for natural deduction systems.

## Conclusions

- Historical and conceptual analysis can unearth some of the connections between the concepts of dialogues and proofs.
- It suggests that every proof can be understood as a ‘dialogue’ between the one formulating it and a built-in opponent: the BIO conception.
- This conception can serve as a starting point to evaluate, from a philosophical perspective, different dialogue-based logical formalisms.
- It offers a possible answer to the Dawkins question for natural deduction systems.

## Conclusions

- Historical and conceptual analysis can unearth some of the connections between the concepts of dialogues and proofs.
- It suggests that every proof can be understood as a ‘dialogue’ between the one formulating it and a built-in opponent: the BIO conception.
- This conception can serve as a starting point to evaluate, from a philosophical perspective, different dialogue-based logical formalisms.
- It offers a possible answer to the Dawkins question for natural deduction systems.



## Conclusions

- Historical and conceptual analysis can unearth some of the connections between the concepts of dialogues and proofs.
- It suggests that every proof can be understood as a ‘dialogue’ between the one formulating it and a built-in opponent: the BIO conception.
- This conception can serve as a starting point to evaluate, from a philosophical perspective, different dialogue-based logical formalisms.
- It offers a possible answer to the Dawkins question for natural deduction systems.