

Remarks on Dialogical Meaning: A Case Study
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Abstract

The dialogical framework is an approach to meaning that provides an alternative to both the model-theoretical and the proof-theoretical semantics.

The dialogical approach to logic is not a logic but a semantic rule-based framework where different logics could be developed, combined or compared. But are there any constraints? Can we introduce rules ad libitum to define whatever logical constant? In the present paper I will explore the first conceptual moves towards the notion of *Dialogical Harmony*.

Crucial for the dialogical approach are the following points

1. The distinction between local (rules for logical constants) and global meaning (included in the structural rules)
2. The player independence of local meaning
3. The distinction between the play level (local winning or winning of a play) and the strategic level (global winning; or existence of a winning strategy).

In order to highlight these specific features of the dialogical approach to meaning I will discuss the dialogical analysis of tonk, some tonk-like operators and the negation of the logic of first-degree entailment .

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S2 Dialogical Logic

In a dialogue two parties argue about a thesis respecting certain fixed rules.

1. The defender of the thesis is called Proponent (**P**), his rival, who attacks the thesis is called Opponent (**O**). In its original form, dialogues were designed in such a way that each of the plays end after a finite number of moves with one player winning, while the other loses.
2. Actions or moves in a dialogue are often understood as *utterances* or as speech-acts. *Declarative utterances* involve formulae; *interrogative utterances* do not involve formulae
3. Moves induce commitments. Commitments are commitments to other moves not to *semantic attributes* such as truth, proof or justification.
4. The rules are divided into particle rules or rules for logical constants (*Partikelregeln*) and structural rules (*Rahmenregeln*). The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves that are challenges (to the moves of a rival) and those moves that are defences (of the player's own moves).

S3

Crucial for the dialogical approach are the following points (that will motivate some discussion further on)

4. The distinction between local (rules for logical constants) and global meaning (included in the structural rules)
5. The symmetry of local meaning
6. The distinction between the play level (local winning or winning of a play) and the strategic level (global winning; or existence of a winning strategy).

S4 Local meaning 1: Particle rules:

In dialogical logic, the particle rules are said to state the *local semantics*: what is at stake is only the challenge and the answer corresponding to the utterance of given logical constant, rather than the whole context where the logical constant is embedded.

$\vee, \wedge, \rightarrow, \neg, \forall, \exists$	Challenge	Defence
X: $A \vee B$	Y: $?-\vee$	X: A <i>or</i> X: B the defender chooses
X: $A \wedge B$	Y: $? \wedge 1$ or Y: $? \wedge 2$ the challenger chooses	X: A <i>respectively</i> X: B
X: $A \rightarrow B$	Y: A	X: B
X: $\neg A$	Y: A	— No defence possible.
X: $\forall x A$	Y: $?-\forall x/k$ challenger chooses	X: $A[x/k]$ For any k chosen earlier by Y
X: $\exists x A$	Y: $? \exists$	X: $A[x/k]$ defender chooses

S5

One interesting way to look at the local meaning is as rendering an abstract view on the logical constants involving the following types of actions:

- a) Choice of declarative utterances (=:disjunction and conjunction).
- b) Choice of interrogative utterances involving individual constants (=: quantifiers).
- c) Switch of the roles of defender and challenger (conditional and negation). As we will discuss later on we might draw a distinction between the switches involved in the local meaning of negation and the conditional).

Let us briefly mention two crucial issues related to the particle rules to which we will come back later on

- **Symmetry:** The particle rules are symmetric in the sense that they are player independent – that is why they are formulated with the help of variables for players. Compare with the rules of tableaux or sequent calculus that are asymmetric: one set of rules for the *true*(left)-side other set of rules for the *false*(right)-side. The symmetry of the particle rules provides, as we will see below, the means to get rid of tonk-like-operators.
- **Sub-formula property:** If the local meaning of a particle # occurring in φ involves *declarative* utterances, these utterances must be constituted by sub-formulae of φ . (This has been pointed out by Laurent Keiff and by Helge Rückert in several communications)

S6 Structural Rules: Global Meaning 1:

(SR 0) (starting rule):

The initial formula is uttered by **P** (if possible). It provides the topic of the argumentation. Moves are alternately uttered by **P** and **O**. Each move that follows the initial formula is either a challenge or a defence.

Comment: The proviso *if possible* relates to the utterance of atomic formulae. See formal rule (SR 2) below.

(SR 1) (no delaying tactics rule):

Both **P** and **O** may only make moves that change the situation.

Comments: This rule should assure that plays are finite (though there might be an infinite number of them). The original formulation of Lorenz made use of ranks, other devices introduced explicit restrictions on repetitions. Ranks, seem to be more compatible with the general aim of the dialogical approach to differentiate between the play level and the strategical level. Let us describe here the rule that implements the use of ranks.

- After the move that sets the thesis players **O** and **P** each chose a natural number n resp. m (termed their repetition ranks). Thereafter the players move alternately, each move being an attack or a defence.
- In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most n (resp. m) times.

S7 Structural Rules: Global Meaning 2:

(SR 2) (*formal rule*): **P** may not utter atomic formulae unless **O** uttered it first. Atomic formulae can not be challenged.

The dialogical framework is flexible enough to define the so-called *material dialogues*, that assume that atomic formulae have a fixed truth-value:

(SR *2) (*rule for material dialogues*):

Only atomic formulae standing for true propositions may be uttered. Atomic formulae standing for false propositions can not be uttered.

(SR 3) (*winning rule*):

X wins iff it is Y's turn but he cannot move (either challenge or defend).

(SR 4i) (*intuitionistic rule*):

In any move, each player may attack a (complex) formula asserted by his partner or he may defend himself against the last attack that has not yet been answered.

or

(SR 4c) (*classical rule*):

In any move, each player may challenge a (complex) formula asserted by his partner or he may defend himself against any attack (including those that have already been defended).

- Notice that the dialogical framework offers a fine-grained answer to the question: Are intuitionist and classical negation the same negations? Namely: The particle rules are the same but it is the global meaning that changes.

S8 Structural Rules: Global Meaning 3

In the dialogical approach validity is defined via the notion of *winning strategy*, where winning strategy for X means that for any choice of moves by Y, X has at least one possible move at disposition such that he (X), wins:

Validity (definition):

A formula is valid in a certain dialogical system iff **P** has a formal winning strategy for this formula.

Thus,

- A is classically valid if there is a winning strategy for **P** in the formal dialogue $Dc(A)$.
- A is intuitionistically valid if there is a winning strategy for **P** in the formal dialogue $Dint(A)$.

S9 Structural Rules: Global Meaning 4

Comments to the formal rule and to validity: Helge Rückert (2011) pointed out, and rightly so, that the formal rule triggers a novel notion of validity.² Validity, is not being understood as being true in every model, but as *having a winning strategy independently of any model* or more generally independently of any *material* grounding claim (such as truth or justification). Copy-cat is not copy cat of groundings but copy-cat of declarative utterances involving atomic formulae. In fact, one could see the formal rule as process the first stage of which starts with what Laurent Keiff called *contentious dialogues*.³ Contentious dialogues are dialogues where a player X utters an atomic formula that is dependent upon a given ground and X is not prepared to put this ground into question – one can think of it as a claim of having some kind of justification (or a claim of truth) for it.⁴ According to Rückert, the formal rule establishes a kind of game where one of the players must play without knowing what the antagonists justifications of the atomic formulae are. Thus, according to this view, the passage to formal dialogues relates to the switch to some kind of games with incomplete information. Now, if the ultimate grounds of a dialogical thesis are atomic formulae and if this is implemented by the use of a formal rule, then the dialogues are in this sense necessarily asymmetric. Indeed, if both contenders were restricted by the formal rule no atomic formula can ever be uttered. Thus, we implement the formal rule by designing one player, called the *proponent*, whose utterances of atomic formulae are, at least, at the start of the dialogue restricted by this rule.

² Talk at the workshop Proofs and Dialogues, Tübingen, Wilhelm-Schickard Institut für Informatik, 25-27; February 2011/.

³ Cf. Clerbout/Keiff/Rahman 209 and in Keiff/Rahman 2010.

⁴ Cf. Keiff/Rahman 2010 (156-157), where this is linked to some specific passages of Plato's *Gorgias* (472b-c).

S10 Structural Rules: Global Meaning 5

Symmetric and asymmetric versions of the intuitionistic structural rule (i)

In the standard literature on dialogues, there is an asymmetric version of the intuitionist rule, called E-rule since Felscher [1985]. It's formulation is the following:

In any move, each player **O** may react only upon the immediately preceding move of **P**.

Now the point of the asymmetric rule is that **O** will never have as his disposition two **P**-formulae to challenge. The symmetric rule on the other hand allows this. If the aim is to produce intuitionist logic, we should implement the rule *last duty first* exactly in those rules that might allow a delay, namely in the conditional and the negation. According to this idea Rahman ([1993]) proposed the following analysis of the role of the E-Rule in intuitionistic logic:

- 1) The asymmetric E-Rule is based on strategic considerations, namely, the different roles in a strategy of the **P**- and the **O**-utterances.
- 2) The symmetric E-Rule is based on meaning considerations, namely the specific local and global meaning of the conditional (and the negation as a special case), that allows locally to switch the roles of challenger and defender and might trigger globally defence delays.
- 3) The asymmetric E-Rule yields a system of strategies that corresponds to Gentzen's Calculus of 1935 (and Kleene 1952), the symmetric E-rule is closer to Beth tableaux (in Rahman (1993), the references have been mistakenly switched. Indeed, the tableaux corresponding to Gentzen 1935 do not allow two formulae to occur at the right side (do not allow that two **P**-formulae occur at the same time in the same branch). Beth tableaux are more permissive.

S11 Structural Rules: Global Meaning 6

Symmetric and asymmetric versions of the intuitionistic structural rule (ii)

- 4) The asymmetric E-Rule allows straightforward proofs of some meta-mathematical properties of intuitionistic logic such as the interpolation theorem and the disjunctive property. For the latter see the following point.
- 5) In the Rahman PHD it is shown how to prove the disjunctive property of intuitionistic logic with the asymmetric E-Rule and it is very briefly mentioned that if in context of the sequent calculus corresponding to the symmetric version; the proof is difficult to carry if we only use the means of sequent calculus. Indeed, without the approach to meaning (that distinguishes between play and strategic levels), typical of dialogical logic, the proof of the disjunctive property is hard to deliver. In his paper *Why Dialogical Logic?* ([2001]) Rückert presents the argument with some detail. The point is that if we consider the distinction between the play and the strategic level then the proof of the disjunctive property can be carried out in the same way with symmetric or asymmetric rules (see appendix 2). A more detailed presentation of the arguments involved have been published before by Rahman/Rückert in 1998 (“Die pragmatischen Sinn und Geltungskriterien der Dialogischen Logik beim Beweis des Adjunktionsatzes”, *Philosophia Scientiae*, 1998-99, vol.3/3, 145-170).

S12 Examples:

Classical and Intuitionistic Structural Rules

In the following dialogue played with classical structural rules **P**' move 4 answers **O**'s challenge in move 1, since **P**, according to the classical rule, is allowed to defend (once more) himself from the challenge in move 1. **P** states his defence in move 4 though, actually **O** did not repeat his challenge – we signalise this fact by inscribing the not repeated challenge between square brackets.

O			P		
				$p \vee \neg p$	0
1	$?_{\vee}$	0		$\neg p$	2
3	p	2		—	
[1]	[$?_{\vee}$]	[0]		p	4

Classical rules. **P** wins.

In the dialogue displayed below about the same thesis as before, **O** wins according to the intuitionistic structural rules because, after the challenger's last attack in move 3, the intuitionistic structural rule forbids **P** to defend himself (once more) from the challenge in move 1.

O			P		
				$p \vee \neg p$	0
1	$?_{\vee}$	0		$\neg p$	2
3	p	2		—	

Intuitionistic rules. **O** wins.

S13 Strategies and Tableaux (i)

Strategies: As mentioned above validity is defined via the notion of *winning strategy*.

If **P** is to win against any choice of **O**, we will have to consider two main different situations, namely

- the dialogical situations in which **O** has uttered a complex formula, and
- those in which **P** has uttered a complex formula.

We call these main situations the **O**-cases and the **P**-cases, respectively. In both of these situations another distinction has to be examined. Namely those cases where **P** chooses and those cases where **O** chooses.

S14 Strategies and Tableaux (ii)

In the standard literature (Lorenzen, Krabbe, Felscher) most descriptions of the available strategies will yield a version of the semantic tableaux where **O** stands for **T** (left-side) and **P** for **F** (right-side) and where situations of type **ii** (and not of type **i**) will lead to a branching-rule.

(P)-Chooses	(O)-Chooses
(P) $\alpha \vee \beta$	(P) $\alpha \wedge \beta$
-----	-----
$\langle \mathbf{O} ? \rangle$ (P) α $\langle \mathbf{O} ? \rangle$ (P) β	$\langle \mathbf{O} ? \wedge 1 \rangle$ (P) α $\langle \mathbf{P} ? \wedge 2 \rangle$ (P) β
<i>The expressions of the form $\langle X \dots \rangle$ constitute interrogative utterances</i>	<i>The expressions of the form $\langle X \dots \rangle$ constitute interrogative utterances</i>
(O) $\alpha \wedge \beta$	(O) $\alpha \vee \beta$
-----	-----
$\langle \mathbf{P} ? \wedge 1 \rangle$ (O) α $\langle \mathbf{P} ? \wedge 2 \rangle$ (O) β	$\langle \mathbf{P} ? \rangle$ (O) α $\langle \mathbf{P} ? \rangle$ (O) β
(P) $\alpha \rightarrow \beta$	(O) $\alpha \rightarrow \beta$
-----	-----
(P) α (O) β	(P) α (O) ? ... (O) β <i>(Opponent has the choice between counterattacking or defending)</i>
<i>No choice</i> (P) $\neg \alpha$	<i>No choice</i> (O) $\neg \alpha$
-----	-----
(O) α	(P) α

S15 Dialogues are not Tableaux

Dialogues are built up bottom up, from local semantics to global semantics and from global semantics to validity. This triggers the priority of the play level over the winning-strategy-level.

The dialogical approach takes the play level as the level where meaning is set and on the basis of which validity rules should result.

The difference between **O (T)**-rules and the **P (F)**-rules of a tableaux is a result of the strategical level and the asymmetry introduced by the formal rule.

S16 Tonk: Tableaux version for tonk:

(O) [(T)] <i>Atonk</i> B	(P) [or (F)] <i>Atonk</i> B
(O) [(T)] <i>B</i>	(P) [(F)] <i>A</i>

From the dialogical point of view, the rejection of tonk is linked to the symmetry condition of the particle rules that cannot be fulfilled for tonk. Indeed; the defence must yield a different formula, namely the tail of tonk if the defender is **O** and the head of tonk if the defender is **P**:

O: <i>Atonk</i> B	P: <i>Atonk</i> B
P: ?	O: ?
O: <i>B</i>	P: <i>A</i>

This means that the attempted particle-rule for tonk is player-dependent, and this should not be the case. The point is that in dialogues tonk-like operators are rejected because there is no symmetric particle rule that justifies the tableaux-rules designed for these operators.

S17 Tunk

(O) [(T)] <i>AtunkB</i> ----- (O) [(T)] <i>A</i> (O) [(T)] <i>B</i>	(P) [or (F)] <i>AtunkB</i> ----- (P) [(F)] <i>A</i> (P) [(F)] <i>B</i>
--	---

Let us attempt to define a player independent particle rule for tunk. Let us thus assume that for a given player **X** that uttered *AtunkB* the challenge (if it should somehow meet the tableaux-rules) must be one of the following:

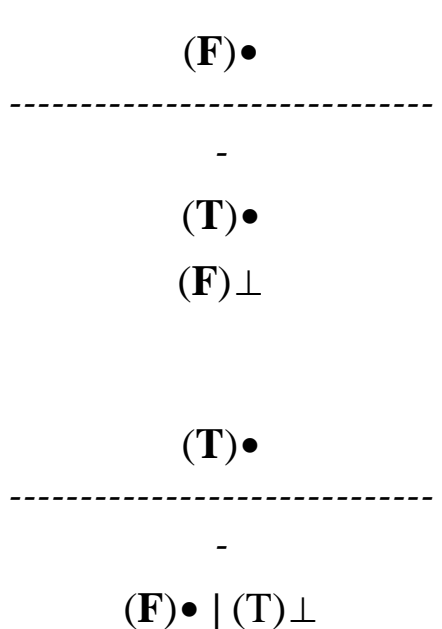
- 1) (**Y**) show me the left side, and (**Y**) show me the right side. Here it is the challenger who has the choice;
- 2) (**Y**) show me at least one of the both sides. Here it is the defender who has the choice.

Now whatever the options are, one of them will clash with one of the tableaux-rules described above:

- If we take option one, the challenger **O** has the choice and this should yield at the *strategic level* a branching on the **P**-rule and no branching on the **O**-Rule.
- . If we take option 2, the defender **O** has the choice and this should produce a branching at the *strategic level* on the **O**-rule and no branching on the **P**-Rule.

S18 Black-Bullet

Stephen Read introduced a different kind of pathological logical constant called *black-bullet*, that can be thought as a kind of a zero-
adic operator and that says of itself that it is false:



S19 Black-Bullet and Dialogues

From the dialogical point of view we can formulate symmetric particle rules for \bullet :

X: \bullet

Y: ?

X: $\neg\bullet$

Furthermore, the dialogical analysis of this particle allows two approaches:

- (i) If we put the emphasis in the fact that \bullet is an operator, then a dialogue with this operator as a thesis will generate an infinite game,
- (ii) if we stick to a semantics that is complete in relation to the tableaux-rules then \bullet has a double nature, namely, on one side it is an operator that can be challenged and on the other side it is an atomic formula and as such should follow the formal rule. This double nature could be rendered by adding a special structural rule like the following:

Black-bullet formal rule: *If O can challenge \bullet iff he has not uttered it before.*

The particle rules for black-bullet make it apparent that \bullet is part of the challenge and defence moves and thus contravenes the sub-formulae property mentioned above.

S20 *Dialogical Harmony (i)*

1. Particle rules must be player-independent
2. Particle rules must fulfil the sub-formula property
3. *(The particle rule of a logical constant must be given independently of the inner structure of the formula in which this logical constant occurs as a main operator.)*
4. Global meaning must be player-independent
5. This assumes that within the structural rules a global meaning. This also assumes that the global meaning does not “undo” the player-independence of the particle rules.
6. Appropriate tableaux systems must be build up bottom up.
7. In other words; those tableaux systems (or sequent calculi), that render a proof theory for a given dialogical semantics must be sound and complete in relation to the latter.

The third condition can be contested as being too strong and is crucial for the discussion of the so-called “dual negation”. In fact, a contravention to the third condition, as will see below does not seem to trigger tonk-like operators.

S21 *Dialogical Harmony (ii)*

Can we establish a kind of dialogical Harmony theorem?

- The particle # is trivializing iff there are no symmetric particle rules for # (with sub-formula property).

Well, what we can do for the moment is to prove the following:

Partial –Dialogical-Harmony-Lemma I (PDL-1) :

(PDL-1.1) If there is a trivializing particle # such that the tableaux-rules –constituted by two lines - (with sub-formula property) -have the following form:

$$\begin{array}{cc}
 (\mathbf{T}) \alpha[\#] & (\mathbf{F}) \alpha[\#] \\
 \hline
 (\mathbf{T}) \beta & (\mathbf{F}) \beta \\
 (\mathbf{T}) \gamma & (\mathbf{F}) \gamma
 \end{array}$$

Then there are no symmetric particle rules (with sub-formula property).

Proof: By contraposition, if there are symmetric particle rules for #, then the tableaux resulting from the winning strategies based on that particle rules do not correspond to the form described above.

Let us start with the case where the tableaux are constituted by two lines:

S22 Dialogical Harmony (iii)

If there were symmetric particle rules for #, then defences and challenges must be player independent.

Let us thus assume that for a given player **X** that uttered $\alpha[\#]$ the challenge (if it should somehow meet the tableaux-rules) must be one of the following:

1) (**Y**) show me the left side, and (**Y**) show me the right side. Here it is the challenger who has the choice;

2) (**Y**) show me at least one of the both sides. Here it is the defender who has the choice.

Now whatever the options are, one of them will clash with one of the tableaux-rules described above when we replace the variables by players:

- If we take option one, the challenger **O** has the choice and this should yield at the *strategic level* a branching on the **P**-rule and no branching on the **O**-Rule.
- If we take option 2, the defender **O** has the choice and this should produce a branching at the *strategic level* on the **O**-rule and no branching on the **P**-Rule.

S23 *Dialogical Harmony (iii)*

The case for one line-tableaux is simpler:

(PDL-1.2) If there is a trivializing particle # such that the tableaux-rules –constituted by one line - have the following form:

$$\begin{array}{cc}
 \text{(T)} \alpha[\#] & \text{(F)} \alpha[\#] \\
 \hline
 \text{(T)} \beta & \text{(F)} \gamma \\
 \text{(where } \beta \text{ is different from } \gamma\text{)} &
 \end{array}$$

Then there are no symmetric particle rules (with sub-formula property).

If there were symmetric particle rules for # then the defence must be constituted by one sole sub-formula that is uttered player-independently and the correspondent tableaux must be then the following

$$\begin{array}{c}
 \text{(O)} \alpha[\#] \\
 \hline
 \text{(O)} \beta \\
 \\
 \text{(P)} \alpha[\#] \\
 \hline
 \text{(P)} \beta
 \end{array}$$

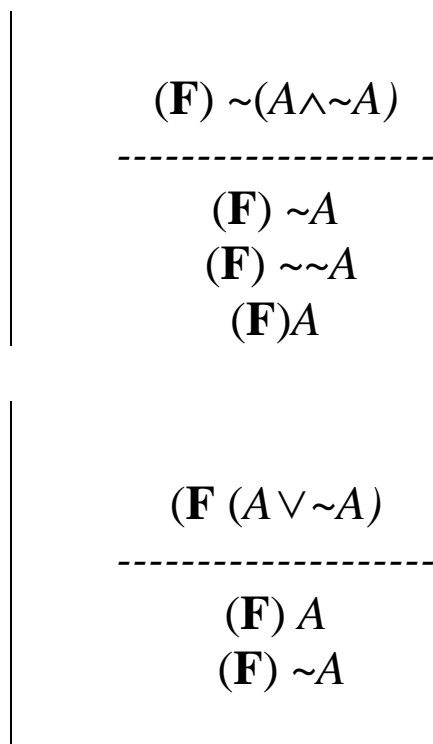
S24 Hintikka's Trees for Enquiry Games 1
(Hintikka/Halonen/Mutanen 1988)

<i>(T)-Cases</i>	<i>(F)-Cases</i>
$(\mathbf{T})\alpha \vee \beta$	$(\mathbf{F})\alpha \vee \beta$
-----	-----
-	
$(\mathbf{T})\alpha \mid (\mathbf{T})\beta$	$(\mathbf{F})\alpha$ $(\mathbf{F})\beta$
$(\mathbf{T})\alpha \wedge \beta$	$(\mathbf{F})\alpha \wedge \beta$
-----	-----
$(\mathbf{T})\alpha$ $(\mathbf{T})\beta$	$(\mathbf{F})\alpha \mid (\mathbf{F})\beta$
$(\mathbf{T})\sim(\alpha \wedge \beta)$	$(\mathbf{F})\sim(\alpha \wedge \beta)$
-----	-----
$(\mathbf{T})\sim\alpha \mid (\mathbf{T})\sim\beta$	$(\mathbf{F})\sim\alpha$ $(\mathbf{F})\sim\beta$
$(\mathbf{T})\sim(\alpha \vee \beta)$	$(\mathbf{F})\sim(\alpha \vee \beta)$
-----	-----
$(\mathbf{T})\sim\alpha$ $(\mathbf{T})\sim\beta$	$(\mathbf{F})\sim\alpha \mid (\mathbf{F})\sim\beta$
$(\mathbf{T})\sim\sim\alpha$	$(\mathbf{F})\sim\sim\alpha$
-----	-----
$(\mathbf{T})\alpha$	$(\mathbf{F})\alpha$

- formulae of the form $(\mathbf{T})A$ and $(\mathbf{F})A$ (for atomic A).
- A Hintikka-tree is closed if all its branches are closed.

S25 Hintikka's Trees for Enquiry Games 2

Examples:



Hintikka describes a tree-system that yields classical logic by adding two extra-closing rules

Namely

if it contains atomic formulae of the form $(\mathbf{T}) \sim A$ and $(\mathbf{T}) A$.

if it contains atomic formulae of the form $(\mathbf{F}) \sim A$ and $(\mathbf{F}) A$.

The first additional rule allows the validity of non-contradiction to be proved

The second additional line allows the validity of third-excluded to be proved

S26 Michael Dunn's relational semantics for FDE

(Dunn 1960)

The idea is that instead of having truth-*functions* truth-*relations* are introduced: allowing a formula to be related to false (0) and true (1) or to neither of them. The fact that a formula α relates to 0 (relates to 0: $\alpha R0$) does not mean that it is untrue, since the formula can also relate to 1 ($\alpha R1$). The fact that a formula does not relate to 1 (it is untrue), does not mean that it relates to 0 (is false) since it might relate with neither. The recursive definitions are the expected ones:

$(\alpha \wedge \beta)R1$ iff $\alpha R1$ and $\beta R1$

$(\alpha \vee \beta)R1$ iff $\alpha R1$ or $\beta R1$

$(\alpha \wedge \beta)R0$ iff $\alpha R0$ or $\beta R0$

$(\alpha \vee \beta)R0$ iff $\alpha R0$ and $\beta R0$

$\sim \alpha R1$ iff $\alpha R0$

$\sim \alpha R0$ iff $\alpha R1$

Semantic consequence is defined in the usual way in terms of truth-preservation, thus

$\Sigma \models \alpha$ iff for every model based on R , if $\beta R1$, for all $\beta \in \Sigma$, then $\alpha R1$.

S27 Negation as Duality 1: Switch of choices

The standard particle rule for negation:

\neg	Challenge	Defence
$X-\neg A$	$Y-A$	— No defence possible

FDE-negation:

\sim	Challenge	Defence
$X-\sim(A \vee B)$	$Y-?\sim \vee_1$ <i>or</i> $Y-?\sim \vee_2$ challenger chooses	$X-!-\sim A$ <i>respectively</i> $X-!-\sim \beta$
$X-\sim(A \wedge B)$	$Y-?\sim \wedge$	$X-!-\sim A$ <i>or</i> $X-!-\sim B$ defender chooses

$\sim\sim$	Challenge	Defence
$X-\sim\sim A$	$Y-?\sim\sim$	$X-A$

S28 Negation as Duality 2: Switch of choices

Particle rule for FDE-negative literals:

The point is that FDE-negation produces a change of choices and since there is no choice to do there is no defensive move possible:

	Challenge	Defence
X - $\sim p$	Y - $? \sim$	— No defence

Formal rule for FDE:

P cannot introduce positive literals: any positive literal must be stated by **O** first.

P can challenge a negative literal iff the same negative literal (uttered by **P**) has been already challenged by **O** before. Positive literals cannot be challenged.

FDE-negation defined by structural rules

P cannot introduce literals: any literal (positive or not) must be uttered by **O** first.

P can utter the double negation of a positive literal if **O** uttered the correspondent negation-free literal before. This double negation utterance of **P** can not be challenged.

P can utter a positive literal if **O** uttered the double negation of the same literal before.

Literals cannot be challenged.

S29 Negation as Duality 3: Examples

O			P		
				$p \vee \sim p$	0
1	$?_{\vee}$	0		$\sim p$	2
3	$?_{\sim}$	2		—	

FDE-rules. **O** wins.

O			P		
				$\sim (p \wedge \sim p)$	0
1	$?_{\sim \wedge}$	0		$\sim \sim p$	2
3	$?_{\sim \sim}$	2			
[1]	$[?_{\sim \wedge}]$	0		$\sim p$	4
5	$?_{\sim}$	4		—	

FDE-rules. **O** wins.

O			P		
H	$\sim p$			$\sim p \vee q$	0
1	$?_{\vee}$	0		$\sim p$	2
3	$?_{\sim}$	2		—	
	—		H	$?_{\sim}$	4

FDE-rules. **P** wins.

S30 CONCLUSIONS

Is the FDE-negation a tonk-like operator? No, it is a logical constant and it allows inconsistency but not triviality.

Local meaning is, according to the dialogical point of view, about symmetry, utterances, how to raise a question in relation to an utterance (*local* challenge) and how to answer to a request (*local* defence). Negation is still switch and FDE-negation seems to stress the point that it is switch of choices. Notice that at one might argue that at the end this in fact what even the structural definition of FDE-negation says. The difference is that in the context of the structural approach the change of choices is the result of a second move.

Perhaps, some might even argue that dual negation represents the core of the dialogical meaning of negation. The switch of challenger and defender roles typical of standard negation might come from the fact that negative literals in some way behave like a conditional.

In other words, meaning in dialogical logic is determined by actions, those actions that set the meaning of negation seem to be linked to a switch:

of defender and challenger roles (standard dialogical negation)
that is linked to the further action of choosing sides (FDE-negation),

Some might argue that switch of choices is typical of negation and switch of challenger-defender roles stems from the conditional. A further discussion of this issue requires an analysis of a conditional compatible with FDE negation. I will leave this for a next paper.