The Conception of Validity in Dialogical Logic

Dr. Helge Rückert
University of Mannheim
Germany

rueckert@rumms.uni-mannheim.de

http://www.phil.uni-mannheim.de/fakul/phil2/rueckert/index.html

Workshop "Proof and Dialogues"
Tübingen
February 2011

Playing chess against Carlsen and Anand

Board 1:

White: Magnus Carlsen (Norway, World No. 1)

Black: Helge (a *patzer*, more or less)

Board 2:

White: Helge

Black: Viswanathan Anand (India, World No. 2)

Helge will score 1/2 against the two best players in the world!

How?

Copycat strategy:

Copy the opponents' moves and make them indirectly play against each other

Dialogical Logic as a Semantic Approach in Logic

Semantic approaches

Denotational/referential approaches (f.e. model theory)	Use-based approaches
A broadly Fregean/Wittgensteinian(I) picture of language and meaning	A broadly Wittgensteinian(II) picture of language and meaning

Use-based semantic approaches

Proof-theoretic approaches (f.e. Natural Deduction)

Game-theoretic approaches (f.e. **Dialogical Logic**)

Rules how to use expressions in proofs

Rules how to use expressions in language games

A very Short Presentation of Dialogical Logic

- Two players, the proponent (**P**) and the opponent (**O**), play a game about a certain formula according to certain rules

- **P** begins with the initial thesis

- The rules are divided into:

  Structural rules
  (they determine the general course of the game)

  Particle rules
  (they determine how formulas, containing the respective particles, can be attacked and defended)

- Each play is won by one player and lost by the other

- Truth is defined in terms of the existence of a winning strategy for **P**

The Particle Rules

	Attack	Defence
¬α	α	⊗ (No defence, only counterattack possible)
α∧β	?L(eft) --------------------- ?R(ight) (The attacker chooses)	α --------------------- β
α∨β	?	α ----------------- β (The defender chooses)
α→β	α	β
∀ρα	$?_c$ (The attacker chooses)	α [c/ρ]
∃ρα	?	α [c/ρ] (The defender chooses)

Remarks:

- The particle rules are player independent
- Attacks and defences are always less complex than the attacked formula
  $\Rightarrow$ Plays unavoidably reach the atomic level

Question: What happens at the atomic level?

Digression: Hintikka’s GTS

Up to this point there are no essential differences between Dialogical Logic and Hintikka’s GTS (Game-Theoretical Semantics).

But:
In GTS the games are always played given a certain model (and the players know about the model!): Atomic formulas are evaluated according to the model and the result of a play can be accordingly determined.

GTS:

- Game-theoretic semantics for the logical connectives
- Model-theoretic semantics for the atoms

$\Rightarrow$ GTS is a combination of a game-theoretic and a model-theoretic approach!

Validity in GTS:
For every model there is a winning strategy (for the first player)

Question:

So, what’s the point of game-theoretic approaches in logic? Isn’t all this just a reformulation of well known things using games talk?

Answer:

Yes, indeed.

So far…

But:

The games approach opens up new possibilities, especially the transition to **games with imperfect or incomplete information**

Digression continued:
Hintikka's Independence Friendly Logic

Main idea:
When concerned with formulas with nested quantifiers, a player having to chose how to attack or defend a quantifier, might lack information about how the other player attacked or defended another quantifier earlier on. In this sense the second quantifier is independent from the first.

Slash notation: $\forall x(\exists y/\forall x)\ R(x,y)$

Then only a uniform strategy for choosing y is possible.

Consequently: $\forall x(\exists y/\forall x)\ R(x,y) \Leftrightarrow \exists y \forall x\ R(x,y)$

But:
The expressive power of IF logic exceeds that of first-order logic.

For example: $\forall x \exists y \forall z(\exists w/\forall x)\ R(x,y,z,w)$

Dialogical Logic and the Formal Rule

What happens at the atomic level in Dialogical Logic?

The distinguishing feature of Dialogical Logic is the so-called formal rule:

Formal rule:
O is allowed to state atomic formulas whenever he wants. **P** is only allowed to state an atomic formula if **O** has stated this atomic formula before

The deeper motivation of this rule can best be explained with a transition to games with incomplete information:

Suppose that **P** lacks information about the atomic level. Let's say that there are rules about how to attack and defend atomic formulas, but **P** doesn't know how they look like. Thus, he also doesn't know which atomic formulas yield a win or a loss.

Two cases:

1) **O** states an atomic formula
   P is unable to attack as he lacks information about how such an attack looks like

2) **P** states an atomic formula
   O attacks it and **P** is unable to react as he lacks information about how a defense looks like

Question:

Is it still possible for **P** to have a winning strategy?

Answer:

Yes! Because of a copycat strategy.

If **O** has already stated an atomic formula before, **P** is safe when stating this atomic formula himself as **O** can’t successfully attack because he then indirectly attacks himself. (If **O** attacks, **P** can copy this attack, and if **O** then defends against the attack, **P** can copy the defense etc etc.) So, in this situation **P** can never loose.

This idea is captured by the formal rule.

Validity in Dialogical Logic

The standard conception (validity as general truth):

Validity as truth in every model

Or: Validity as the existence of a winning strategy given any model

The dialogical conception (validity as formal truth):

Validity as the existence of a winning strategy despite lacking information about the atomic level

Or: Validity as the existence of a winning strategy when the formal rule is in effect

The Conception of Meaning in Dialogical Logic

- Particle rules

  $\Rightarrow$ Meaning of the logical connectives
  (local meaning)
  How to attack and defend

- Particle rules + structural rules (without the formal rule)

  $\Rightarrow$ Meaning of propositions
  (global meaning)
  How to play games

- Formal rule

  $\Rightarrow$ Making the plays independent of the
  meaning of the atoms
  (transition to logic!)

Plays vs. Strategies

- Level of **plays**

  $\Rightarrow$ Game rules

  (How to play?)

  Meaning is constituted by the game rules

- Level of **strategies**

  $\Rightarrow$ Strategic rules

  (How to play well? Does a winning strategy exist?)

  Concepts like **truth** and **validity** are defined at the level of strategies

Strategic Tableaux

- Procedure to determine for which formulas there exists a winning strategy
- They result from the level of plays

(O)-*cases*	**(P)-*cases***
$(\mathbf{O})\alpha\vee\beta$ ---------------------------------- $\langle(\mathbf{P})?\rangle\ (\mathbf{O})\alpha \mid \langle(\mathbf{P})?\rangle\ (\mathbf{O})\beta$	$(\mathbf{P})\ \alpha\vee\beta$ -------------------- $\langle(\mathbf{O})?\rangle\ (\mathbf{P})\alpha,\ \langle(\mathbf{O})?\rangle\ (\mathbf{P})\beta$
$(\mathbf{O})\alpha\wedge\beta$ --------------------- $\langle(\mathbf{P})?L\rangle\ (\mathbf{O})\alpha,\ \langle(\mathbf{P})?R\rangle\ (\mathbf{O})\beta$	$(\mathbf{P})\alpha\wedge\beta$ ---------------------------------- $\langle(\mathbf{O})?L\rangle\ (\mathbf{P})\alpha \mid \langle(\mathbf{O})?R\rangle\ (\mathbf{P})\beta$
$(\mathbf{O})\alpha\rightarrow\beta$ ------------------------------- $(\mathbf{P})\alpha,\ \ldots \mid \langle(\mathbf{P})\alpha\rangle\ (\mathbf{O})\beta$	$(\mathbf{P})\alpha\rightarrow\beta$ ------------------- $(\mathbf{O})\alpha,\ (\mathbf{P})\beta$
$(\mathbf{O})\neg\alpha$ ------------------ $(\mathbf{P})\alpha,\ \langle\otimes\rangle$	$(\mathbf{P})\neg\alpha$ --------------- $(\mathbf{O})\alpha,\ \langle\otimes\rangle$
$(\mathbf{O})\forall\rho\alpha$ -------------------- $\langle(\mathbf{P})?_c\rangle\ (\mathbf{O})\alpha[c/\rho]$ (c does not need to be new)	$(\mathbf{P})\forall\rho\alpha$ -------------------- $\langle(\mathbf{O})?_c\rangle\ (\mathbf{P})\alpha[c/\rho]$ (c is new)
$(\mathbf{O})\exists\rho\alpha$ -------------------- $\langle(\mathbf{P})?\rangle\ (\mathbf{O})\alpha[c/\rho]$ (c is new)	$(\mathbf{P})\exists\rho\alpha$ -------------------- $\langle(\mathbf{O})?\rangle\ (\mathbf{P})\alpha[c/\rho]$ (c does not need to be new)

Concluding Remarks:
Proofs and Dialogues

- Dialogical Logic is NOT a proof-theoretic approach

- A dialogue is NOT a proof

- In a dialogue **P** does NOT try to prove the initial formula

- If **P** wins he has NOT proved the initial formula